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Nonlinearity in the theory of hot electrons due to excess of charge carriers in the presence of electric field

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Abstract
The excess of nonequilibrium charge carriers due to heating by strong electric fields influences the nonlinear current–voltage characteristics in semiconductors substantially. It is shown that in general the electron gas heating results in a change of the carrier concentration in the conduction band due to the dependence of the recombination rate on the electron temperature. This nonequilibrium carrier density leads to an extra nonlinearity coefficient in the current–voltage characteristic curve even when the electron mobility is independent of the electron temperature. These latter contributions in the nonlinear theory of hot electrons are exemplified by an attractive potential of the impurity potential in semiconductors.

As electronic systems are used more and more widely in almost all areas of endeavor and daily life, the semiconductor technology on which they are based is being pushed to ever larger scale integration and ever greater miniaturization. As the devices get smaller, and smaller, new problems appear.

One consequence of the reduction in size is that the electric fields accelerating electrons and holes through the crystal become very large, so that the carriers acquire large kinetic energies. Strong electric fields produce a great variety of effects in semiconductors [1]. They alter basically the quantum states of carriers, their energy spectrum and the equilibrium carrier concentration. This gives rise to dependences of the macroscopic properties of semiconductors on the applied field E. Examples of such effects include: the deviation from Ohm's law of the current–voltage characteristics—in this situation the carrier mobility begins to increase or decrease with the increase of the electric field; hot electron diffusion and invalidation of the Einstein relation; a process of recombination of impurity levels in semiconductors; the dependence of the complex dielectric constant on E, resulting from the possibility of fundamental absorption of photons whose energy is less than the forbidden band gap (the Franz–Keldysh effect).

The classical theory of hot carrier transport in semiconductors has been discussed in a number of books [2–5] and review articles [6–9]. Previous calculations have been addressed to nonlinearity caused by impact ionization [1, 10], intervalley distribution of carriers [11] or a nonparabolic carrier energy dispersion law [12, 13]. More recently, nonlinear transport in narrow gap semiconductors due to an excess of nonequilibrium electron gas due to impact ionization and the Auger recombination process as the basic mechanism for carrier recombination and generation have become well known from a series of earlier papers [14–16], within the framework of the balance equation approach.

Investigation of the effect of an electric field on the thermal ionization and recombination of carriers has recently been used to obtain the multiphonon parameters of deep-level impurities in semiconductors [17].

In the single-valley semiconductor approximation, neglecting the effects on the nonlinearity of the current density mentioned above, previous theories usually assume that only the thermal equilibrium density of carriers is subjected to heating by electric fields, i.e. in the heating process the concentration of carriers remains constant. Therefore, the nonlinear current–voltage characteristic is only related to the electron mobility through the electron temperature.

Recently, the role of taking into consideration the change in densities of electrons and holes in both the conduction and the valence band due to heating by the electric field has been considered in the nonlinear behavior of the current density in semiconductors [18, 19]. Physically, this effect can be understood as follows: in the presence of an electric field, the carriers acquire energy from the field and lose it by different mechanisms (phonon, impurity, surface scattering, etc). In quasielastic collisions of electrons or holes with phonons or other quasiparticles, this extra energy of the carriers is greater, on average, than what they have at thermal equilibrium. As the field increases, the average energy of electrons and holes also increases, and they can be described using an effective temperature T_{n} for electrons and T_{p} for the holes in the valence band which are higher than the lattice temperature T_{ph}. In this case electrons and holes tend to occupy higher quantum states in the conduction and valence bands respectively, and as a consequence carrier capture by band to band recombination or recombination by an impurity level depends strongly on the effective hot carrier temperature and, therefore, the balance between thermal generation and recombination of electrons and holes is destroyed. The difference in temperature between hot electrons and holes causes a change in the carrier concentration because of electron–hole pair recombination in the presence of a heating electric field. The rise in carrier density leads, in turn, to an extra contribution to the nonlinear term in the current density as compared with previous theories [19].

It is well known that in hot transport reported in the literature, the current–voltage characteristics can be superlinear or sublinear with the electric field; however, in this work it is shown that the nonlinear dependence of the current density can change from sublinear to superlinear characteristic behavior. Besides that, if only the electron density is changed due to heating, the hole charge density also changes; this fact enhances the nonlinear current–voltage characteristics.

One of the most representative materials for studying the effect of this new mechanism on hot electrons is a semiconductor with a gap energy $\leqslant 1$ eV, where the recombination rate of electrons and holes as a function of the average energy is more significant.

In [19], we showed that the nonequilibrium electron and hole densities in terms of the carrier capture coefficient and its derivative may be written as

$$\delta n = -\frac{a_p b_t + a_t b_p}{a_p b_n + a_n b_p}\delta T_{\mathrm{n}}, \qquad \delta p = \frac{a_t b_n - a_n b_t}{a_p b_n + a_n b_p}\delta T_{\mathrm{n}}, \tag{1}$$

where $\delta T_{\mathrm{n}} = T_{\mathrm{n}} - T_0$, T_0 is the ambient temperature and

$$\begin{aligned}
a_n &= \alpha_n(T_0)\,(n_0 + n_1 + N_t - n_{\mathrm{t}}(T_0)) + \alpha_p(p_0 + p_1) \\
a_t &= n_0(N_t - n_{\mathrm{t}}(T_0)) \left.\frac{\partial \alpha_n}{\partial T_{\mathrm{n}}}\right|_{T_{\mathrm{n}}=T_0} \\
a_p &= \alpha_n(T_0)(n_0 + n_1) + \alpha_p(p_0 + p_1 + n_{\mathrm{t}}(T_0)) \\
b_n &= \frac{N_t - n_{\mathrm{t}}(T_0)}{n_0 + n_1} \\
b_t &= \frac{n_0(N_t - n_{\mathrm{t}}(T_0))}{p_0 + p_1} \frac{1}{\alpha_n(T_0)} \left.\frac{\partial \alpha_n}{\partial T_{\mathrm{n}}}\right|_{T_{\mathrm{n}}=T_0} \\
b_p &= \frac{n_{\mathrm{t}}(T_0)}{p_0 + p_1}.
\end{aligned} \tag{2}$$

Here n_0 (p_0) is the electron (hole) equilibrium density, n (p) is the electron (hole) nonequilibrium concentration, n_{t} represents the concentration of electrons trapped by the impurity centers of density N_t, $n_1 = \nu_n(T_0)\mathrm{e}^{-(\varepsilon_{\mathrm{c}}-\varepsilon_t)/T_0}$, $p_1 = \nu_p(T_0)\mathrm{e}^{-(\varepsilon_{\mathrm{c}}-\varepsilon_t-\varepsilon_g)/T_0}$ being the concentrations of electrons and holes thermally excited to the conduction and valence bands respectively, when the chemical potential coincides with the impurity level ε_t;

$$\nu_n(T_0) = \frac{1}{4}\left(\frac{2m_{\mathrm{n}}T_0}{\pi\eta^2}\right)^{3/2}, \qquad \nu_p(T_0) = \frac{1}{4}\left(\frac{2m_{\mathrm{p}}T_0}{\pi\eta^2}\right)^{3/2} \tag{3}$$

and m_{n} and m_{p} are the electron and hole effective masses. In equations (1) and (2) we have assumed that $T_{\mathrm{p}} = T_{\mathrm{ph}} = T_0$ (strong electron–phonon energy interaction).

Let us note that the excess of carrier concentration in the presence of heating electric fields is caused mainly by the change in the rate of carrier recombination via trapping levels rather than thermal excitation, i.e. by definition the factor for capture of electrons by the trapping centers can be written as

$$\alpha_n(T_{\mathrm{n}}) = \frac{\int_{\varepsilon_{\mathrm{c}}}^{\infty} c_n(\varepsilon) D_n(\varepsilon) f_n(\varepsilon, T_{\mathrm{n}})\,\mathrm{d}\varepsilon}{\int_{\varepsilon_{\mathrm{c}}}^{\infty} D_n(\varepsilon) f_n(\varepsilon, T_{\mathrm{n}})\,\mathrm{d}\varepsilon} \tag{4}$$

and due to the strong hole–phonon energy interaction, the factor for capture of holes $\alpha_p(T)$ is evaluated at $T = T_0$. Here $c_n(\varepsilon)$ is the probability of an electron transition between the impurity level and the conduction band with energy ε, $D_n(\varepsilon)$ is the electronic density of states, $f_n(\varepsilon, T_{\mathrm{n}})$ is the Maxwell distribution function for electrons in the conduction band at temperature T_{n} and ε_{c} is the energy of an electron at the bottom of the conduction band.

If the momentum of the electron is dissipated by only one type of mechanism, the momentum relaxation time is given by $\tau_n(T_{\mathrm{n}}) = \tau_0(T_{\mathrm{n}}/T_0)^{q_n}$, where q_n is a parameter of the order of unity and its numerical value depends on whether the electron is scattered by phonons or impurities. Then, the current density $\boldsymbol{j} = \boldsymbol{j}_n + \boldsymbol{j}_p$ is related to the heating electric field $\boldsymbol{E}$ by

$$\begin{aligned}
J = J_n + J_p &= \frac{ne^2\tau_n(T_{\mathrm{n}})}{m_n}E + \frac{pe^2\tau_p(T_0)}{m_{\mathrm{p}}}E \\
&= \sigma_n^0\left(1 + \frac{q_n}{T_0}\delta T_{\mathrm{n}} + \frac{\delta n}{n_0}\right)E + \sigma_p^0\left(1 + \frac{\delta p}{p_0}\right)E.
\end{aligned} \tag{5}$$

It is worth mentioning that in previous theoretical models on hot carrier transport $\delta n = \delta p = 0$; therefore, as can be observed from equation (5) the excess of nonequilibrium carrier density due to heating of the hole and electron gas by the electric field introduces a new contribution to the current–voltage characteristics which mainly comes from the dependence of the electron trapping probability on the electron temperature (see equations (1), (2)).

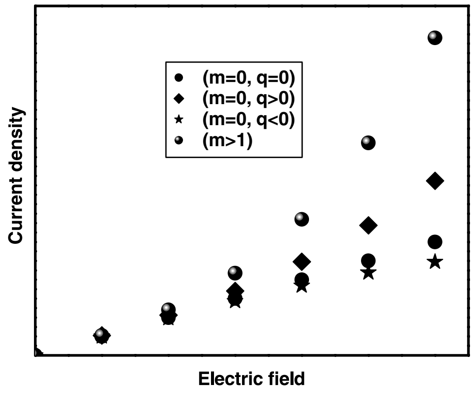

Figure 1. Schematic representation of the current–voltage characteristics. As can be observed, when the variation of the carrier density due to the heating by strong electric field is taken into account, the current density is superlinear independently of the value of q.

For simplicity and in order to illustrate the effect of heating of electron and holes by the electric field on the hot carrier transport, we restrict ourselves to analyzing equation (5) for an n-type semiconductor with a simple attractive impurity potential [20]. In this case $\alpha_n(T_\mathrm{n}) \propto T_\mathrm{n}^{-m}$ where m varies in the range $1 < m < 5$ depending on the nature of the semiconductor and impurities; then,

$$\frac{\delta T_\mathrm{n}}{\alpha_n(T_0)} \frac{\partial \alpha_n}{\partial T_\mathrm{n}}\bigg|_{T_\mathrm{n}=T_0} = -m\frac{\delta T_\mathrm{n}}{T_0}. \tag{6}$$

Since the equilibrium concentration of electrons in the conduction band is given as $n_0 = \nu_n(T_0)\mathrm{e}^{-(\varepsilon_\mathrm{c}-\varepsilon_\mathrm{f})/T_0}$, where ε_f is the Fermi energy, we may write the ratio of $n_\mathrm{t} = N_t(1 + 0.5\mathrm{e}^{(\varepsilon_t-\varepsilon_\mathrm{f})})^{-1}$, the number of electrons associated with unionized donors, to $n_0 + n_\mathrm{t}$, the total number of free and loosely bound electrons, as

$$\frac{n_\mathrm{t}}{n_0 + n_\mathrm{t}} = \frac{1}{1 + \frac{\nu_n}{2N_t}\mathrm{e}^{-(\varepsilon_\mathrm{c}-\varepsilon_t)}} = \frac{1}{1 + \frac{n_1}{2N_t}}. \tag{7}$$

Now $\varepsilon_\mathrm{c} - \varepsilon_t$ is the donor ionization energy, which is ordinarily of the order of or smaller than T_0; the exponential factor above is therefore of the order of unity. If $N_t \ll \nu_n(T_0)/2$ the ratio of the number of electrons on unionized donors to the total number will be very small (at room temperature ν_n is of the order of 10^{19} cm^{-3}). In such instances, it is clear that the donors are completely ionized, and it is usually convenient and accurate to proceed on the assumption that their ionization is compete, setting $n_\mathrm{t} = 0$. At very low temperatures the exponential factor in equation (6) may be appreciably smaller than unity, and the criterion for complete ionization must be written in the more general form $N_t \ll n_1/2$. Under this approximation equation (5) acquires the form $J = \sigma^0(1 + \beta E^2)E$ where β is the nonlinear coefficient which describes the influence of the carrier heating on the electrical conductivity and is given by

$$\beta = \frac{\sigma_n^0/n_0\nu_\varepsilon}{\sigma^0}\left[\sigma_n^0\left(\frac{q_n}{T_0} + \frac{n_0+n_1}{p_0}\frac{m}{T_0}\right) + \sigma_p^0\frac{n_0(n_0+n_1)}{p_0^2}\frac{m}{T_0}\right] \tag{8}$$

with $\sigma^0 = \sigma_n^0 + \sigma_p^0$. Because $|q_n| \sim 1$ and $(n_0 + n_1) \gg p_0$, this implies that the nonlinear current–voltage characteristic is independent of how the momentum electron is dissipated and therefore the current density for n-type semiconductor has a superlinear dependence on the heating electric field. It is important to remark that the nonlinearity in equation (5) is determined only by the dependence of the carrier capture factor on the electron temperature T_n; this remarkable contribution is negligible in standard theories of hot electron transport.

In figure 1, we show a schematic representation of the nonlinearity of the current density as a function of the electric field. It is important to note that when the variation of the nonequilibrium carrier density due to the heating of carriers by the electric field is neglected, i.e. $m = 0$, the current–voltage characteristics reported in standard theories on hot carrier transport are reproduced; in other words, one obtains the linear ($q_n = 0$), sublinear ($q_n < 0$) and the superlinear ($q_n > 0$) stages. On the other hand, when the dependence of the electron trapping probability on the electron temperature is considered, the current–voltage characteristic is always superlinear since $|q_n| \sim 1$ and $1 < m < 5$, i.e. the nonlinear current density is independent of how the momentum is dissipated.

Thus, in conclusion, the electron concentration changes caused by the carrier heating alter the kinetic coefficients. In classical theory of hot carrier transport, the excess of carrier concentration is usually neglected and the current–voltage characteristic is linear when $q_n = 0$; in this work and with a simple example, we showed that $\delta n \neq 0$ nonequilibrium charge carriers due to heating by a strong electric field contribute an extra term to the nonlinearity current–voltage characteristics even for the case where the electron is scattered by neutral impurities or the polarization potential of acoustic phonons at low temperature ($q_n = 0$).

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